REAL NUMBERS

Euclid's Division Lemma

Statement

Given positive integers 'a' & 'b', there exist a unique integers 'q' and 'r' satisfying

$$a = b \times q + r$$

where $0 \leq r < b$

Euclid's Division Algorithm

$c = d \times q + r$

~~Statement~~

→ An Application of Euclid's Division Lemma

→ Helps in finding HCF of numbers

* Algorithm

It is a series of well defined steps which gives a procedure for solving a type of problem.

* Lemma

It is a proven statement used for proving another statement

Date
Page

① Find the HCF of 60 & 36 using EDA.

By EDA, $a = bq + r$

→ $60 = 36 \times 1 + 24 \quad (r \neq 0)$

$36 = 24 \times 1 + 12 \quad (r \neq 0)$

$24 = 12 \times 2 + 0 \quad (r = 0)$

$\therefore$ HCF = 12

② Find the HCF of 24 & 15 using EDA.

→ By EDA,

$a = bq + r$

$24 = 15 \times 1 + 9 \quad (r \neq 0)$

$15 = 9 \times 1 + 6 \quad (r \neq 0)$

$9 = 6 \times 1 + 3 \quad (r \neq 0)$

$6 = 3 \times 2 + 0 \quad (r = 0)$

$\therefore$ HCF = 3

③ Find the HCF of 8 & 6.

→ By EDA,

$8 = 6 \times 1 + 2 \quad (r \neq 0)$

$6 = 2 \times 3 + 0 \quad (r = 0)$

$\therefore$ HCF = 2

(4) Find the HCF of 135 & 225.

→ By EDA,

$225 = 135 \times 1 + 90 \quad (r \neq 0)$
$135 = 90 \times 1 + 45 \quad (r \neq 0)$
$90 = 45 \times 2 + 0 \quad (r = 0)$

$\therefore$ HCF = 45

(5) Find the HCF of 867 & 255.

→ By EDA,

~~255~~ $867 = 255 \times 3 + 102 \quad (r \neq 0)$
$255 = 102 \times 2 + 51 \quad (r \neq 0)$
$102 = 51 \times 2 + 0 \quad (r = 0)$

$\therefore$ HCF = 51

(6) Find the HCF of 616 & 32.

→ $616 = 32 \times 19 + 8 \quad (r \neq 0)$
$32 = 8 \times 4 + 0 \quad (r = 0)$

$\therefore$ HCF = 8

(7) Prove that every positive even integer is of the form $2q$ & every positive odd integer is of the form $2q+1$, where q is some integer.

→ Let a be any +ve integer & $b = 2$
($a = bq + r$)

Apply EDA to 'a' & 'b'

$$a = 2q + r \quad , \quad 0 \leq r < 2$$

$$\text{so } r = 0, 1$$

$$a = 2q \longrightarrow \text{Even}$$
$$a = 2q+1 \longrightarrow \text{Odd}$$

Every composite no. can be factorised as a product of prime nos in a unique way (apart from order)

(8) Can 4^n end with a zero? (Where 'n' is a natural no.)

→ $4^n = 2^{2n}$

It cannot end with zero as it has only factor 2.

(9) Find LCM & HCF of 6 & 20 by method of prime factorisation.

→ $6 = 2 \times 3$
$20 = 2 \times 2 \times 5$

* HCF is the product of smallest powers of each common prime factor of the number.

$\therefore HCF = 2$

* LCM is the product of the greatest power of each prime factor involved of the numbers.

$\therefore LCM = 2 \times 2 \times 3 \times 5$
$= 60$

$HCF(a, b) \times LCM(a, b) = a \times b$ (Product of two no.s)

$HCF \times LCM =$ Product of two number

(10) Find the HCF of 96 and 404 by prime factorisation and use it to find their LCM.

→ $96 = 32 \times 3$
$= 2^5 \times 3$

$404 = 2^2 \times 101$

$\therefore HCF = 2^2 = 4$

$HCF \times LCM = \text{Product of two numbers}$

$2^2 \times LCM = 96 \times 404$

$$LCM = \frac{96 \times 404}{4}$$

$LCM = 9696$

(11) Find the LCM & HCF of 12, 15, 21 by prime factorisation.

→ $12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$HCF = 3$

$LCM = 2^2 \times 3 \times 5 \times 7$

$= 420$

(12) Find LCM & HCF of 92 & 510 by prime factorisation

→ $92 = 2^2 \times 23$

$510 = 2 \times 3 \times 5 \times 17$

So $HCF = 2$

$LCM = 2^2 \times 3 \times 5 \times 17 \times 23$

$= 23460$

* Irrational Number

A number is called irrational, if it cannot be expressed as p/q, where 'p' & 'q' are integers and $q \neq 0$

\# Theorem :- If 'p' be a prime number

If 'p' divides 'a^2', then 'p' will divide 'a' also, 'a' is a positive integer.

(13) Prove that $\sqrt{2}$ is irrational.

→ By using Method of Contradiction

Assume $\sqrt{2}$ is a rational number.

$\sqrt{2} = a/b \quad (b \neq 0)$

$b\sqrt{2} = a \quad$ (a & b are co-primes)
(HCF as 1)

Squaring both the sides

$(b\sqrt{2})^2 = a^2$

$2b^2 = a^2 \quad$ (2 divides a)

Let $a = 2k \quad$ (k is an integer)

$2b^2 = 4k^2$

$b^2 = 2k^2 \quad$ (2 divides b)

∴ a & b have 2 as a common factor

∴ Our assumption is wrong.

Hence, $\sqrt{2}$ is an irrational number

* Any real number which has a decimal expansion that terminates can be expressed as a rational number whose denominator is a power of 10.

* Rational number = p/q

where, $q = 2^n \times 5^m$ and 'n', 'm' are non-negative integers

* If 'x' is a rational number whose decimal expansion terminates, then 'x' can be written in the form p/q (p, q are co-primes) and the prime factorisation of 'q' will be of the form $2^n \times 5^m$, where 'n' & 'm' are non-negative integers.

* If the rational no. $x = p/q$ ($q \neq 0$; p, q are co-primes) such that the prime factorisation of 'q' is of the form of $2^n \times 5^m$ or either only 2^n or 5^m where 'n' & 'm' are non-negative integers, then 'x' will have decimal expansion which terminates.

* The square of natural number can never end in 2, 3, 7, or 8.

If $q = 2^n \times 5^m$ the decimal expansion will terminate.

For eg. $= 3/8 = 3/2^3 = 0.375$

$1/8 = 1/2^3 = 0.125$

$875/10000 = 875/2^4 \times 5^4 = 0.0875$

If $q \neq 2^n \times 5^m$ the decimal expansion will not terminate repeating.

For e.g. $= 1/7 = 0.\overline{142857}$

(14) Without ~~po~~ using long division, check if the given rational numbers are terminating or recurring.

(a) $13/3125 = p/q$

To verify $= 3125 = 2^n \times 5^m$

$q = 3125 = 5^5 \times 2^0$

$\therefore$ Its terminating

(15) The least number must be added to 1056, so that the sum is completely divisible by 23 is 2.

We have,

$$\begin{array}{r} 23)\,1056\,(45 \\ 92 \\ \hline 136 \\ 115 \\ \hline 21 \\ \hline \end{array}$$

Require no. $= 23 - 21$

$= 2$